Instructions

1. The theoretical competition will be 5 hours in duration and is marked out of a total of 300 points.

2. There are **Answer Sheets** for carrying out detailed work/rough work. On each **Answer Sheet**, please fill in

- Student Code
- Question No.
- Page no. and total number of pages.

3. Start each problem on a separate Answer Sheet. Please write only on the printed side of the sheet. Do not use the reverse side. If you have written something on any sheet which you do not want to be marked, cross it out.

4. Please remember that the graders may not understand your language. As far as possible, write your solutions only using mathematical expressions and numbers. If it is necessary to explain something in words, please use short phrases (if possible in English).

5. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, etc.), please put up your hand to signal the proctor.

6. The beginning and end of the competition will be indicated by a long sound signal. Additionally, there will be a short sound signal fifteen minutes before the end of the competition (before the final long sound signal).

7. At the end of the competition you must stop writing immediately. Sort and put your sheets in

separate stacks,

(a) Stack 1: answer sheets of part 1
(b) Stack 2: answer sheets of part 2
(c) Stack 3: answer sheets of part 3
(d) Stack 4: question papers and paper sheets that you do not want to be graded.

8. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

9. A list of constants for this competition is given on the next page.

Point distribution of this exam

Problem number	Points
T1	10
T2	10
T3	10
T4	10
T5	10
T6	25
T7	25
T8	25
T9	25
T10	75
T11	75
Total	300

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	$9.81\ \mathrm{ms}^{-2}$	
Obliquity of Ecliptic	23°27′	**Earth**
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass ($M_☾$)	7.35×10^{22} kg	
Radius ($R_☾$)	1.74×10^{6} m	
Mean Earth-Moon distance	3.84×10^{8} m	**Moon**
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	−12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	**Sun**
Absolute Magnitude	4.80 mag	
Surface Temperature	5772 K	
Angular diameter at Earth	30′	
Orbital velocity in Galaxy	$220\ \mathrm{kms}^{-1}$	
Distance from Galactic center	8.5 kpc	
1 *au*	1.50×10^{11} m	
1 *pc*	206265 au	
Gravitational constant (G)	$6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$	
Planck constant (h)	6.62×10^{-34} Js	
Boltzmann constant (k_B)	$1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$	
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	
Hubble constant (H_0)	$67.8\ \mathrm{kms^{-1}Mpc^{-1}}$	**Physical constants**
Speed of light in vacuum (c)	$299792458\ \mathrm{ms}^{-1}$	
Magnetic Permeability of free space (μ_0)	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$	
1 Jansky (Jy)	$10^{-26}\ \mathrm{W\ m^{-2}Hz^{-1}}$	

Rayleigh-Jeans law is given by $B_\nu = \frac{2k_BT}{c^2}\nu^2$, which is the power emitted per unit emitting area, per steradian, per unit frequency.

(T1) Super Luminal Galaxies **(10 points)**

Read the statements given below and state if they are true or false:

(a) For some galaxies the apparent recession speed exceeds the speed of light.

(b) The velocity – Distance relation as given by Hubble cannot allow recession velocities to exceed the speed of light.

(c) Hubble-Lemaitre's law (formerly known as Hubble's Law) does not violate special relativity.

(d) If some galaxies would have an apparent recession speed exceeding the speed of light, then the photons from those galaxies can never reach us.

(e) As the expansion of Universe is accelerating, photons emitted right now from galaxies which have apparent recession speed equal to the speed of light will never reach us.

(T2) Distance **(10 points)**

An observer measured trigonometric parallaxes of stars in a star cluster. Due to random errors, the measured parallax values are distributed symmetrically around the expected value with standard deviation equal to 0.05 mas (milliarcsec). Assume there are no systematic errors and assume all stars in the said cluster have the same luminosity. It is known that the distance of this cluster from us is R= 5 kpc.

He gave the data table to 4 of his students (A, B, C and D) and they estimated the distance to the cluster in the following ways:

A. Convert each parallax measurement into distance and then find the average distance (R_A)
B. Take the average of all parallaxes first and then convert the average parallax into distance. (R_B)

C. Convert each parallax measurement into distance and then take the median distance value. (R_C)

D. Find the median value of the parallaxes and then convert the median value into distance. (R_D)

State if the following statements are true or false. **In case a given mathematical relation is false, give the correct relation.**

(l) If the i^{th} star gave the smallest value of parallax and the j^{th} star gave the highest value of parallax, in all likelihood R_i-R > R-R_j

(m) R_A = R (i.e. there is a high chance that the distance estimated by A fairly matches the true distance)

(n) R_B = R (i.e. there is a high chance that the distance estimated by B fairly matches the true distance)

(o) R_C < R (i.e. there is a high chance that the distance estimated by C will be systematically lower than the true distance)

(p) R_D = R (i.e. there is a high chance that the distance estimated by D fairly matches the true distance)

(T3) Atmospheric Refraction **(10 points)**

Consider sunrise at Beijing (φ=40°) on the vernal equinox day.

(a) Let us say r_l, r_d, r_r and r_u are distances from the centre of the undistorted disk of the Sun to the edge of the disk towards the directions left, down, right and up respectively. What will be the hierarchical relation (<, =, >) between the four radii just after the sunrise?

(b) What is the correction in the time of rise of the top edge of the disk as compared to the case without atmosphere? You may assume that typically atmospheric refraction near the horizon is 35'. Please only consider the apparent diurnal motion.

(T4) Height of a Hill **(10 points)**

Two friends wanted to measure the height of the hill next to their village (latitude $\varphi=40°$). One of the friends climbed to the top of the hill and she agreed to send a light signal to her friend in the village as soon as she sees the sunset. On March 21, when they did this experiment, the friend in village received the light signal 4.1 minutes after the sunset from the village. Estimate the height of the hill and horizon distance for the person at the hill top. Ignore atmospheric refraction.

(T5) Sidereal Time **(10 points)**

It is very interesting to observe that on one particular calendar day each year, the mean sidereal time will twice be 00:00:00.

(a) What will be the approximate R.A. of the Sun when this event happens?

(b) Estimate the exact date in 2018 for this event.

You may assume that at the Royal Greenwich Observatory, the mean sidereal time ($GMST_0$) was 6.706h at 0h, 1st January, 2018 (JD2458119.5).

(T6) Observe the Sun with FAST **(25 points)**

The Five-hundred-meter Aperture Spherical radio Telescope (FAST) is a single-dish radio telescope located in Guizhou Province, China. The physical diameter of the dish is 500 m, but during observations, the effective diameter of the collecting area is 300 m.

Consider observations of the thermal radio emission from the photosphere of the Sun at 3.0 GHz with this telescope and a receiver with bandwidth 0.3 GHz.

(a) Calculate the total energy ($E_\odot$) that the receiver will collect during 1 hour of observation.
(b) Estimate the energy needed to turn over one page of your answer sheet (E'). Hint: the typical surface density of paper is 80 gm^{-2}.
(c) Which one is larger?

(T7) Sunspot **(25 points)**

Magnetic fields are important in the physics of stars and sunspots. As an approximation, we can model the photosphere of the Sun consisting of a plasma, which can be simply treated as a single component ideal gas, and a magnetic field (**B**), which has an associated magnetic pressure $p_B = \frac{B^2}{2\mu_0}$. It behaves like any other physical pressure except that it is carried by the magnetic field rather than by the kinetic energy of particles.

Assume that the number density of particles in the photosphere is constant everywhere, but the magnetic field inside the sunspot (B_{in}=0.1T) is much stronger than outside ($B_{out}=5\times10^{-3}$T). From the blackbody spectrum, the temperature inside the sunspot is T_{in}~4000K, while the temperature outside is T_{out}~6000K (which is why the sunspot looks darker). For the sunspot to be stable, the inside must be in equilibrium with the outside.

(a) Estimate the number density of plasma particles in the solar photosphere.

(b) Compare your answer with an estimate of the number density of particles in the atmosphere at the surface of the Earth.

(T8) A Possible Dark Matter Deficient Galaxy **(25 points)**

Earlier this year, a team of astronomers reported their discovery of a galaxy with much less dark matter than the galaxy evolution model predicted (van Dokkum et al. 2018, Nature). This galaxy, named NGC 1052-DF2, is located close to the elliptical galaxy NGC 1052 (D=20Mpc from the Sun) in the sky. The shape of NGC 1052-DF2 resembles an ellipse with semi major axis (*a*) of 22.6" and $\frac{b}{a} = 0.85$. Half of the total light from the galaxy comes from within this ellipse and the mean surface brightness within the ellipse is about 24.7 mag arcsec^{-2}.

(a) Calculate the total apparent magnitude of this galaxy.

(b) The team suggested the galaxy is a companion of NGC 1052. Determine the total mass of stars in NGC 1052-DF2, assuming it has a mass to light ratio $\left(\frac{M/M_\odot}{L/L_\odot}\right)$ of 2.0.

(c) The team identified 10 globular clusters in NGC 1052-DF2 with a mean galactocentric distance of 78.4". They also measured the velocity dispersion of these clusters to be not more than 8.4 km/s. Estimate the dynamical mass of this galaxy. For simplicity, assume the mass distribution in the galaxies is uniform and is spherically symmetric.

(d) This discovery was challenged by other groups (Kroupa et al., Nature, 2018, Truijlo et al., MNRAS, 2018), who claimed that NGC 1052-DF2 is not a satellite of NGC 1052, and it is located at a much smaller distance to us. Show why a smaller distance would weaken the assertion of the dark matter deficiency in NGC 1052-DF2.

(T9) Radio Galaxy **(25 points)**

An observer wants to use the Five-hundred-meter Aperture Spherical radio Telescope (FAST) in China to observe a radio galaxy at redshift of $z = 0.06$. We assume that the radio source is compact compared to the beam size of the telescope at the observing frequencies, i.e., the source is point-like as seen through the telescope. To detect a point source with FAST, it must be sufficiently strong (bright) relative to the noise level (for single polarization observations), σ, which depends on the bandwidth, $\Delta\nu$, and the integration time (the radio astronomy equivalent of exposure time), t_i, as follows:

$$\sigma = \frac{2k_B T_{sys}}{A_e\sqrt{t_i \Delta\nu}}$$

where T_{sys} is the system temperature (about 150 K in the frequency range of 0.28 GHz – 0.56 GHz and 25 K in the frequency range of 1.05 GHz – 1.45 GHz), and A_e = 4.6x10^4 m^2 is the effective area of the telescope taking into account the total efficiency of the instrument.

This radio galaxy has an observed continuum flux density of $f_\nu = 2.5 \times 10^{-3} Jy$ at an observing frequency of 0.4 GHz. The bandwidth $\Delta\nu$ for the continuum observation centered at 0.4 GHz is 2.8×10^8 Hz.

(a) In order to detect the continuum flux density at 0.4 GHz with a signal-to-noise ratio of 30 (a so-called 30σ detection), what is the required integration time, t_i?

(b) We want to search for the neutral Hydrogen (HI) in the galaxy using 21cm absorption line. The HI 21cm line, with rest frame frequency of 1.4204 GHz. Calculate the observed frequency (υ_{obs}) of the HI line for this galaxy.

(c) The radio continuum emission from this galaxy can be described by a power law $f_\nu \sim \nu^\alpha$, with a spectral index of $\alpha = -0.2$. Calculate the continuum flux density at ν_{obs} for this galaxy.

(d) The line width of the HI 21cm absorption line is 90 km/s. Calculate the line width in Hz at the observing frequency of ν_{obs}. According to Figure 1, the HI 21cm line absorbs 4% of the continuum flux density (on average) over the line width of 90 kms^{-1}. In order to detect the absorption line at ≥3σ in three consecutive 30 kms^{-1} channels, what is the required integration time?

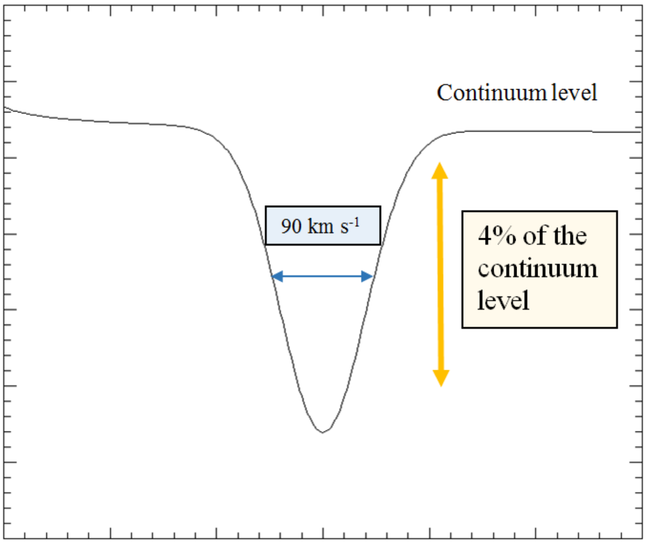

Figure 1: Spectrum of the HI 21cm absorption relative to the continuum emission in the radio galaxy

(T10) Vega and Altair **(75 points)**

As per a very famous Chinese folklore about love, Vega and Altair are two lovers. It is said that they can meet each other once every year on a bridge made up of birds over the Milky Way. The parameters of two stars are given in the table below. For the purpose of this question, assume that the coordinate frame is fixed (i.e. not affected by precession or motion of the Sun).

Star	Right Ascension (J2000.0)	Declination (J2000.0)	Parallax (mas)	Proper Motion		Radial Velocity (km/s)
				$\mu_\alpha \cos\delta$ (mas/year)	μ_δ (mas/year)	
Vega	$18^h36^m56.49^s$	+38° 47' 07.7"	130.23	+200.94	+286.23	-13.9
Altair	$19^h50^m47.70^s$	+8° 52' 13.3"	194.95	+536.23	+385.29	-26.1

Based on this data, answer the following questions:
(a) (9 points) What is the angular separation of the two stars?
(b) (6 points) Calculate the distance (in parsecs) between Vega and Altair.
(c) (3 points) Calculate position angles of the proper motion vectors of each of these two stars.

For parts d-g, assume that the angular velocity of the stars on the celestial sphere remains constant. This is not a physical situation but this is an assumption to simplify the problem.
(d) (2 points) How many common points on the celestial sphere are there which can be reached by both these stars?
(e) (20 points) Find the coordinates of the closest such point.
(Note: Drawing the situation on a celestial sphere will help you in visualising the situation)
(f) (8 points) Find when (which year) each of these stars were / will be at that point.
(g) (5 points) When Altair was / will be at that point, what would be its angular separation from Vega?
(h) (22 points) Find coordinates of any point (if it exists) in 3-D space which was /will be visited by both these stars. Do not ignore radial velocities for this part of the question.

(T11) Thermal History of the Universe **(75 points)**

Based on Einstein's general relativity, Russian physicist Alexander Friedmann derived the Friedmann Equation by which the dynamics of a homogeneous and isotropic universe can be well described. The Friedmann Equation is usually written as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}.$$

We define the Hubble parameter as $= \frac{\dot{a}}{a}$, where a is the scale factor and $\dot{a}$ is the rate of change of scale factor with time. Thus, the Hubble parameter is a function of cosmic time. In the Friedmann Equation, ρ_m is the density of matter, including dark matter and baryons, ρ_r is the density of radiation, Λ is the cosmological constant, and k is the curvature of space. Subscript 0 indicates the value of a physical quantity at present day, e.g. H_0 is the present value Hubble parameter. Also, to avoid confusion with the reduced Hubble parameter, we use the reduced Planck Constant $\hbar = h/(2\pi)$ instead of the Planck constant h.

(a) (5 points) What are the dimensions of Hubble parameter? One can define a characteristic timescale for the expansion of the Universe (i.e. Hubble time t_H) using the Hubble parameter. Calculate the present-day Hubble time t_{H0}.

(b) (5 points) Let us define the critical density ρ_c as the matter density required to explain the expansion of a flat universe without any radiation or dark energy. Find an expression of the critical density, in terms H and G. Calculate the present critical density ρ_{c0}.

(c) (6 points) It is convenient to define all density parameters in a dimensionless manner like $\Omega_i = \frac{\rho_i}{\rho_c}$, i.e. the ratio of density to critical density. The Friedmann Equation can be rewritten using these dimensionless density parameters simply as, $\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1$.

Use this information to find expression for Ω_Λ and Ω_k, in terms H, c, Λ, k and a.

(d) (7 points) Another equation which is valid for matter, radiation and dark energy is often called the Fluid Equation: $\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$, where p is the pressure of some component, ρ is the density and $\dot{\rho}$ is the rate of change of density over time. Radiation contains photons and massless neutrinos, and they both travel at the speed of light. The pressure exerted by these particles is 1/3 of their energy density. Show that the density of radiation $\rho_r \propto (1+z)^4$, where z is cosmological redshift. You may note that if $\frac{\dot{\rho}}{\rho} = n\frac{\dot{a}}{a}$, then $\rho \propto a^n$

(e) (4 points) We know that the value of the cosmological constant Λ doesn't evolve. Its equation of state has a form $p = w\rho_\Lambda c^2$, where w is an integer. Find the value of w.

(f) (13 points) Planck time, defines a characteristic timescale before which our present physical laws are no longer valid, and where quantum gravity is needed. The expression for Planck time can be written in terms of $\hbar$, G and c and non-dimensional coefficient of this expression in SI units is of the order of unity. Using dimensional analysis, find expression for Planck time and estimate its value.

(g) (7 points) Planck length defines the length scale associated with Planck time is given by $l_P = ct_P$. The minimal mass of a black hole, also called Planck mass, is defined as the mass of a black hole whose Schwarzschild radius is two times the Planck length.

Derive the Planck mass M_P and calculate $M_P c^2$ in GeV. This mass is considered to be an upper threshold for elementary particles, beyond which they will collapse to a black hole.

(h) (4 points) At the very beginning (soon after the Planck time), all the particles were in thermal equilibrium in a primordial soup. As temperature decreased, different particles then decoupled from the primordial soup one by one and could travel freely in the Universe. Photons decoupled at ~300000 years after the Big Bang. These photons emitted at that time are what constitutes the cosmic microwave background (CMB), which follows the Stefan-Boltzmann law for blackbody radiation.

$$\varepsilon_r = \frac{\pi^2}{15\hbar^3 c^3}(k_B T)^4,$$

Show that the temperature of the CMB follows $T/(1+z) = constant$.

(i) (16 points) With the expansion of the Universe, radiation density dropped more quickly than matter density, and at some epoch the matter density was equal to the radiation density. Radiation contains both photons and neutrinos. Apart from photons, neutrinos additionally contribute to the radiation energy density by 68% (i.e. $\Omega_{r0} = 1.68\Omega_{\gamma 0}$, where γ indicates photons). Estimate the redshift of matter-radiation equality z_{eq} in terms of Ω_{m0} and reduced Hubble parameter $h = \frac{H_0}{100km\, s^{-1} Mpc^{-1}}$ You may use the current temperature of the CMB: $T_0 = 2.73K$.

(j) (8 points) The neutrinos decoupled from the primordial soup when the temperature of the universe was around 1 MeV. At this time, the radiation density in the universe was much more than all other components. Estimate the time ($t = \frac{1}{2H}$) when neutrinos decoupled, and express it in seconds since the big bang.

Instructions

1. The data analysis competition lasts for 5 hours and is worth a total of 150 points.

2. Dedicated IOAA **Summary Answer Sheets** are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding **Summary Answer Sheet**. On each Answer Sheet, please fill in

- Student's Code

3. **Graph Paper** is required for your solutions. On each Graph Paper sheet, please fill in

- Student's Code
- Question no.
- Graph no. and total number of graph paper sheets used.

4. There are **Answer Sheets** for carrying out detailed work/rough work. On each Answer Sheet, please fill in

- Student's Code
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5. Start each problem on a separate Answer Sheet. **Please write only on the printed side of the sheet. Do not use the reverse side.** If you have written something on any sheet which you do not want to be marked, cross it out.

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9. At the end of the competition you must stop writing immediately. Sort and put your **Summary Answer Sheets**, **Graph Papers**, and **Answer Sheets in one stack**. Put all other papers in another stack. You are not allowed to take any sheet of paper out of the examination area.

10. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

11. A list of constants is given on the next page.

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
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Length of Tropical Year	365.2422 mean solar days	
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Albedo	0.39	
Mass ($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
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Albedo	0.14	
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Planck constant (h)	6.62×10^{-34} Js	
Boltzmann constant (k_B)	1.38×10^{-23} JK^{-1}	
Stefan-Boltzmann constant (σ)	5.67×10^{-8} Wm^{-2}K^{-4}	
Hubble constant (H_0)	67.8 kms^{-1}Mpc^{-1}	
Speed of light in vacuum (c)	299792458 ms^{-1}	
Magnetic Permeability of free space (μ_0)	$4\pi\times10^{-7}$ Hm^{-1}	
1 Jansky (Jy)	10^{-26} W m^{-2}Hz^{-1}	

$$\Delta \log_{10}(x) = \frac{\Delta x}{x\, ln10}$$

(D1) Dust and Young Stars in Star-forming Galaxies **[75 points]**

As a by-product of the star-forming process in a galaxy, interstellar dust can significantly absorb stellar light in ultraviolet (UV) and optical bands, and then re-emit in far-infrared (FIR), which corresponds to a wavelength range of 10-300 μm.

1.1. In the UV spectrum of a galaxy, the major contribution is from the light of the young stellar population generated in recent star-formation processes, thus the UV luminosity can act as a reliable tracer of the star-formation rate (SFR) of a galaxy. Since the observed UV luminosity is strongly affected by dust attenuation, extragalactic astronomers define an index called the *UV continuum slope* (β) to quantify the shape of the UV continuum:

$$f_\lambda = Q \cdot \lambda^\beta$$

where f_λ is the monochromatic flux of the galaxy at a given wavelength λ (in the unit of $\mathrm{W\,m^{-3}}$) and Q is a scaling constant.

(D1.1.1) (6 points) AB magnitude is a specific magnitude system. The AB magnitude is defined as:

$$\mathrm{m_{AB}} = -2.5 \log \frac{f_\nu}{3631\ \mathrm{Jy}}$$

The AB magnitude of a typical galaxy is roughly constant in the UV band. What is the **UV continuum slope** of this kind of galaxy? (Hint: $f_\nu \Delta\nu = f_\lambda \Delta\lambda$)

(D1.1.2) (12 points) Table 1 presents the observed IR photometry results for a $z = 6.60$ galaxy called *CR7*. **Plot** the AB magnitude of *CR7* **versus the logarithm of the rest-frame** wavelength on graph paper and labelled as **Figure 1**.

(D1.1.3) (5 points) Calculate *CR7*'s UV slope, **plot** the best-fit UV continuum on *Figure 1* and make a comparison with the results you obtained in (D1.1.1). Is it dustier than the typical galaxy in (D1.1.1)? **Please answer with [YES] or [NO]**. (Hint: Express $\mathrm{m_{AB}}$ as a function of λ and m_{1600} , where m_{1600} is the AB magnitude at $\lambda_0 = 160$ nm (1600 Å))

Table 1. (Observed Frame) IR Photometry of CR7 at z = 6.60

Band	*Y*	*J*	*H*	*K*
Central Wavelength (μm)	1.05	1.25	1.65	2.15
AB Magnitude	24.71 ± 0.11	24.63 ± 0.13	25.08 ± 0.14	25.15 ±0.15

1.2. Under the assumption that dust grains in the galaxy absorb the energy of UV photons and re-emit it by blackbody radiation, the relation between the UV continuum slope (β), UV brightness (at 1600 Å) and FIR brightness could be established:

$$\mathrm{IRX} \equiv \log\left(\frac{F_{FIR}}{F_{1600}}\right) = S(\beta)$$

where F_{FIR} is the observed far-infrared flux and F_{1600} is the observed flux at rest-frame wavelength 160 nm (1600 Å) (The "flux" F_λ is **defined** as $F_\lambda = \lambda \cdot f_\lambda$). Table 2 presents 20 measurements of β, F_{FIR} and F_{1600} in nearby galaxies (Meurer et al. 1999).

Table 2. UV slope, flux and FIR flux of 20 nearby galaxies

Galaxy Name	UV Slope β	$\log(F_{1600}/10^{-3}\text{Wm}^{-2})$	$\log(F_{FIR}/10^{-3}\text{Wm}^{-2})$
NGC4861	-2.46	-9.89	-9.97
Mrk 153	-2.41	-10.37	-10.92
Tol 1924-416	-2.12	-10.05	-10.17
UGC 9560	-2.02	-10.38	-10.41
NGC 3991	-1.91	-10.14	-9.80
Mrk 357	-1.80	-10.58	-10.37
Mrk 36	-1.72	-10.68	-10.94
NGC 4670	-1.65	-10.02	-9.85
NGC 3125	-1.49	-10.19	-9.64
UGC 3838	-1.41	-10.81	-10.55
NGC 7250	-1.33	-10.23	-9.77
NGC 7714	-1.23	-10.16	-9.32
NGC 3049	-1.14	-10.69	-9.84
NGC 3310	-1.05	-9.84	-8.83
NGC 2782	-0.90	-10.50	-9.33
NGC 1614	-0.76	-10.91	-8.84
NGC 6052	-0.72	-10.62	-9.48
NGC 3504	-0.56	-10.41	-8.96
NGC 4194	-0.26	-10.62	-8.99
NGC 3256	0.16	-10.32	-8.44

(D1.2.1) (14 points) Based on the data given in Table 2, **plot** the IRX $-\ \beta$ diagram on graph paper and labelled as **Figure 2** and find a linear fit to the data. **Write down** your best-fit equation (i.e. $\text{IRX} = a \cdot \beta + b$) by the side of your diagram.

(D1.2.2) (6 points) Quantify the **dispersion** (in 'units' of dex, where **for example,** $\mathbf{\log(10^9) - \log(10^4) = 5\ dex}$) between the observed IRX_{obs} and predicted IRX_{pred} using the following equation:

$$\sigma = \sqrt{\frac{\sum(\Delta\text{IRX}_i)^2}{N-1}}\ (\text{unit: dex})\ where\ \Delta\text{IRX}_i = \text{IRX}_{i,\text{obs}} - \text{IRX}_{i,\text{pred}}$$

1.3. Under the previous assumption of the energy transfer process, the ratio of F_{FIR} to F_{1600} can be expressed as:

$$\frac{F_{FIR}}{F_{1600}} \approx 10^{0.4A_{1600}} - 1$$

Where F_{1600} is the unattenuated flux and A_λ is the dust absorption (in magnitudes) as a function of wavelength λ.

(D1.3.1) (6 points) Express A_{1600} as a function of IRX.

(D1.3.2) (12 points) Based on Table 2 data and the function of $A_{1600}(IRX)$ you derived above, **plot** the $A_{1600} - \beta$ diagram on graph paper and label it as **Figure 3** and find a linear fit to the data. **Write down** your best-fit equation (i.e.$A_{1600} = a' \cdot \beta + b'$) by the side of your diagram.

(D1.3.3) (2 points) If your linear model in (D1.3.2) is correct, what is the expected **UV continuum slope** β_0 of a dust-free galaxy?

1.4. After establishing the local relation between UV continuum slope and IRX, we could probably test this empirical law in the high-redshift universe. In 2016, researchers obtained an Atacama Large Millimeter / submillimeter Array (ALMA) observation of CR7, and the FIR continuum corresponded to a 3σ upper limit of an FIR flux of 1.5×10^{-19} W m^{-2}.

(D1.4.1) (6 points) Calculate the **IRX of *CR7***. Is it an upper limit or lower limit?

Hint: here F_{1600} should be written in the form of:

$$F_{1600} = \lambda_0 \cdot f_{1600}$$

where $\lambda_0 = 160$ nm (1600 Å) and f_{1600} is the observed flux in the rest-frame

(D1.4.2) (6 points) Is the current observation long enough to show any deviation of *CR7* from the IRX$-\beta$ relationship you just derived in the local universe? **Please answer with [YES] or [NO] on the summary answer sheet, give the IRX difference and show the working used to calculate it on the answer sheet**.

(D2) Compact Object in a Binary System [75 points]

Astronomers discovered an extraordinary binary system in the constellation of Auriga during the course of the Apache Point Observatory Galactic Evolution Experiment (APOGEE). In these questions, you will try to analyse the data and recreate their discovery for yourself.

The research team is aiming to find compact stars in binary systems using the radial velocity (RV) technique. They examined archival APOGEE spectra of "single" stars and measured the apparent variation of their RV within this data. Among ~200 stars with the highest accelerations, researchers searched for periodic photometric variations in data from the All-Sky Automated Survey for Supernovae (ASAS-SN) that might be indicative of transits, ellipsoidal variations or starspots. After this process, they spotted a star named 2M05215658+4359220, with a large variation in RV and photometric variability.

2.1. The following table presents the radial velocity measurements of 2M05215658+4359220 during three epochs of APOGEE spectroscopic observation. Here we assume the variation of its RV is due to the existence of an unseen companion. The proper motion of the stars can be ignored.

Table 3. APOGEE Radial Velocity Measurements of 2M05215658+4359220

Observation	MJD	RV ($km\ s^{-1}$)	Uncertainty ($km\ s^{-1}$)
1	56204.9537	-37.417	0.011
2	56229.9213	34.846	0.010
3	56233.8732	42.567	0.010

(D2.1.1) (6 points) Use the data and a simple linear model to obtain an initial estimate of the apparent **maximum acceleration** of the star**:**

$$a_{max} = \frac{\Delta RV}{\Delta t}\bigg|_{max}, \text{ unit: km s}^{-1}\text{ day}^{-1}$$

(D2.1.2) (9 points) Now use the data to obtain an initial estimate of the **mass** of its unseen companion.

2.2. After discovering this peculiar star, astronomers conducted follow-up observations using the 1.5-m Tillinghast Reflector Echelle Spectrograph (TRES) at the Fred Lawrence Whipple Observatory (FLWO) located on Mt. Hopkins in Arizona, USA. The following table presents the RV measurements using this instrument:

Table 4. TRES Radial Velocity Measurements of 2M05215658+4359220

MJD	RV (km/s)	Uncertainty (km/s)
58006.9760	0	0.075
58023.9823	-43.313	0.075
58039.9004	-27.963	0.045
58051.9851	10.928	0.118
58070.9964	43.782	0.075
58099.8073	-30.033	0.054
58106.9178	-42.872	0.135
58112.8188	-44.863	0.088
58123.7971	-25.810	0.115
58136.6004	15.691	0.146
58143.7844	34.281	0.087

(D2.2.1) (14 points) **Plot** the diagram of RV variation (measured with TRES) versus time on your graph paper and label it as Figure 4. Draw a suitable **sinusoidal function** to fit the given data. **Estimate** the orbital period (P_{orb}) and radial velocity semi-amplitude (K) from your plot.

(D2.2.2) (4 points) If the star is moving in a circular orbit, calculate the **minimum value** of the orbital radius (r_{orb}) of the star in units of both $R_{\odot}$ and au.

(D2.2.3) (7 points) The mass function of a binary system is defined as:

$$f(M_1, M_2) = \frac{(M_2 \sin i_{orb})^3}{(M_1 + M_2)^2}$$

where the subscript "1" represents the primary star and "2" represents its companion. The parameter i_{orb} is the orbital inclination of the binary system. This mass function can also be expressed in terms of observable parameters. Calculate the **mass function of this system** in units of $M_{\odot}$.

2.3. Based on a detailed analysis on APOGEE, TRES spectra and GAIA parallax measurements, astronomers derived the following stellar parameters:

Table 5. Selected Physical Properties of 2M05215658+4359220

Effective Temperature T_{eff} (K)	Surface Gravity $\log g$ (cm s^{-2})	Parallax π (mas)	Measured Rotation Velocity $v_{rot} \sin i$ (km s^{-1})	Bolometric Flux F (J s^{-1} m^{-2})
4890 ± 130	2.2 ± 0.1	0.272±0.049	14.1±0.6	$(1.1\pm0.1)\times10^{-12}$

Photometric observations indicate that the period of its light curve is identical to its orbital period, thus we may assume that the rotation period satisfies $P_{rot} = P_{orb} \equiv P$, and the inclination satisfies $i_{orb} = i_{rot} \equiv i$.

(D2.3.1) (16 points) **Calculate** the luminosity (L_1, in unit of $L_{\odot}$), radius (R_1, in unit of $R_{\odot}$), sine of the inclination angle ($\sin i$), as well as mass (M_1, in unit of $M_{\odot}$) of the visible star. Please **include** the uncertainty in your results.

(D2.3.2) (4 points) **Choose** the correct type of this star from the following options: (1) Blue Giant (2) Yellow main sequence star (3) Red Giant (4) Red main sequence star (5) White Dwarf.

(D2.3.3) (10 points) Based on the mass function $f(M_1, M_2)$ of the binary system, **plot** the rough relationship of M_2 (as vertical axis) and M_1 (as horizontal axis) on your graph paper and label it as Figure 5. Plot the most probable relation (by using $\sin i$), upper limit (with $\sin i + \Delta \sin i$) and lower limit (with $\sin i - \Delta \sin i$) derived in (D2.3.1).

(D2.3.4) (5 points) Draw a vertical **shadowed region** of $[M_1 - \Delta M_1, M_1 + \Delta M_1]$, as well as two horizontal **dashed lines** showing the maximum mass of the white dwarf and neutron star, on your *Figure 5*. What is the possible mass of the invisible companion, and what kind of celestial object could it be?

Instructions

1. This part of the contest consists of 4 problems, is 1 hour long and is worth a total of 100 points.
2. Only blue pen should be used to fill in the answer boxes, draw and mark on the star chart.
3. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, etc.), please put up your hand to signal the supervisor.
4. The beginning and end of the competition will be indicated by a long sound signal.
5. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition area.

O1 **(40 points)**

Figure 1 is a whole sky star chart of Yanqing, Beijing at 20:30 tonight (UTC+8) with the limit magnitude = 5^m (m = magnitude). Four stars (about 1^m - 3^m) and one planet (brighter than 2^m) are missing in this chart. In the chart, the distance from the centre is in proportion to zenith distance

(1) (20 points) Draw a cross (X) on the location of each missing star and mark "T" on the chart, and draw a cross (X) on the location of the missing planet and mark "P" on the chart.

(2) (5 points) Please mark the orientation of the star chart with "N" "E" "S" "W" at the edge of the star chart.

(3) (10 points) On the chart, the celestial equator passes through many constellations. Please write down the name of any five of these constellations (IAU codes).

Answer:

(4) (5 points) Using the star chart, estimate the altitude of Aldebaran (α Tau), to the nearest degree.

Answer:

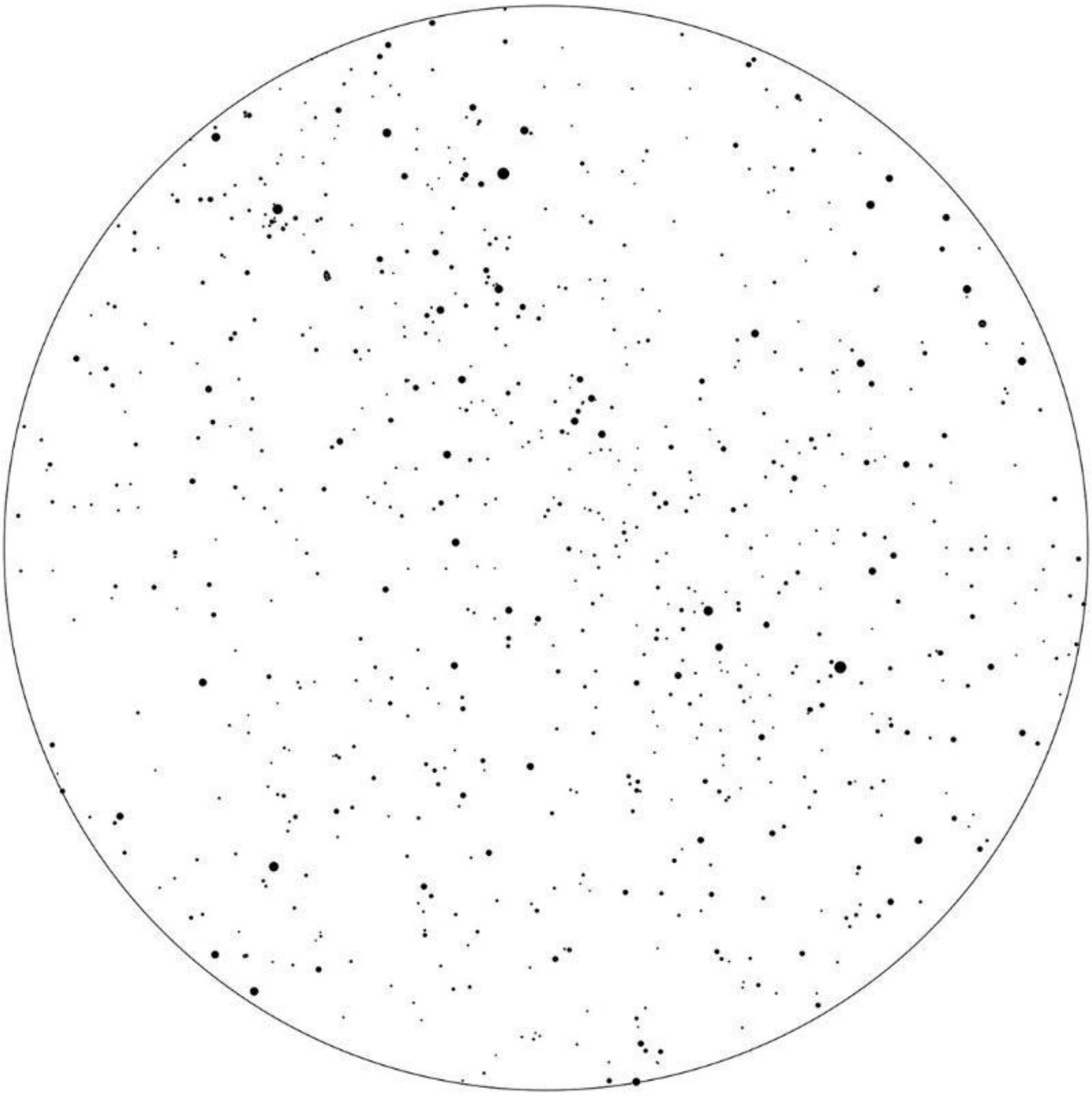

Figure 1

O2 **(20 points)**

Figure 2 is a star chart of a recent opposition of Jupiter. The grid in the figure is the ecliptic coordinates. Please estimate the date of this opposition, to the nearest day.

Answer:

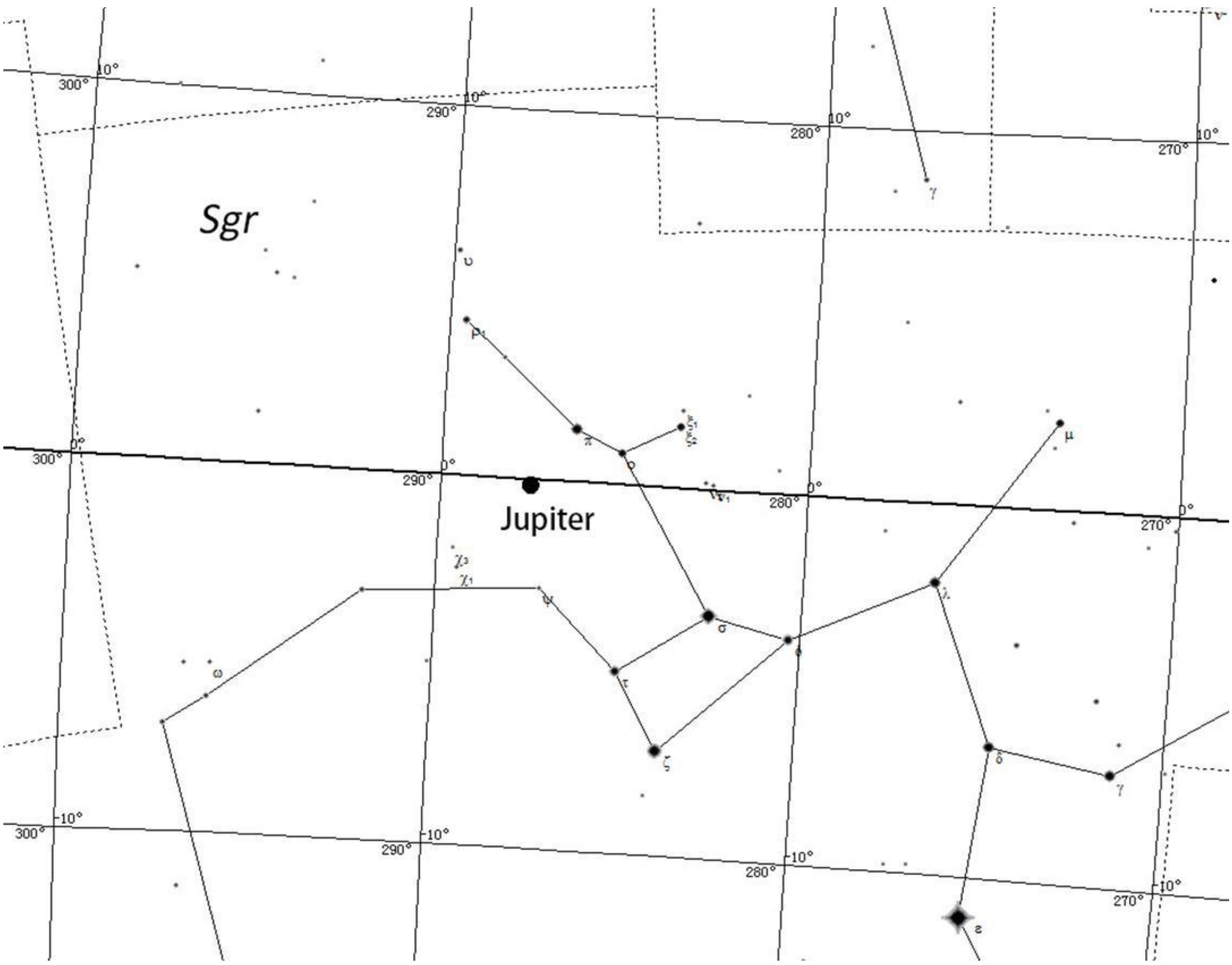

Figure 2

O3 **(20 points)**

Figure 3 is a star chart of a part of the sky on March 21, 2018. The longitude and latitude of the observation site is 120 °E, 40 °N (UTC+8). The grid in the figure is an equatorial grid. The thicker vertical line in the centre is the meridian. Estimate the mean solar time to an accuracy of better than 0.5h.

Answer：

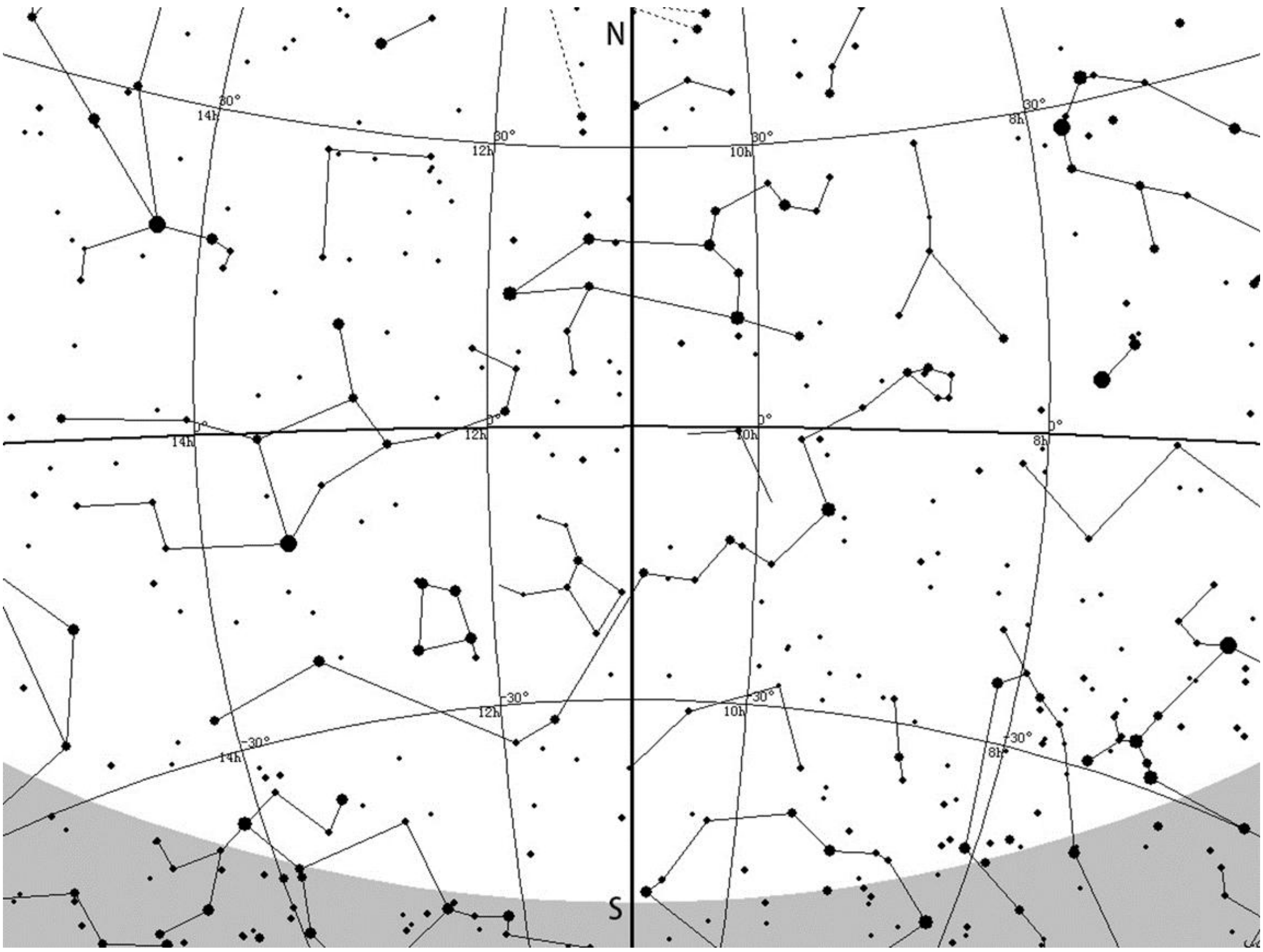

Figure 3

O4 **(20 points)**

Figures 4.1 – 4.4 are four photos of Messier objects. For each of them, please write down the Messier catalogue number and name the constellation where it is located.

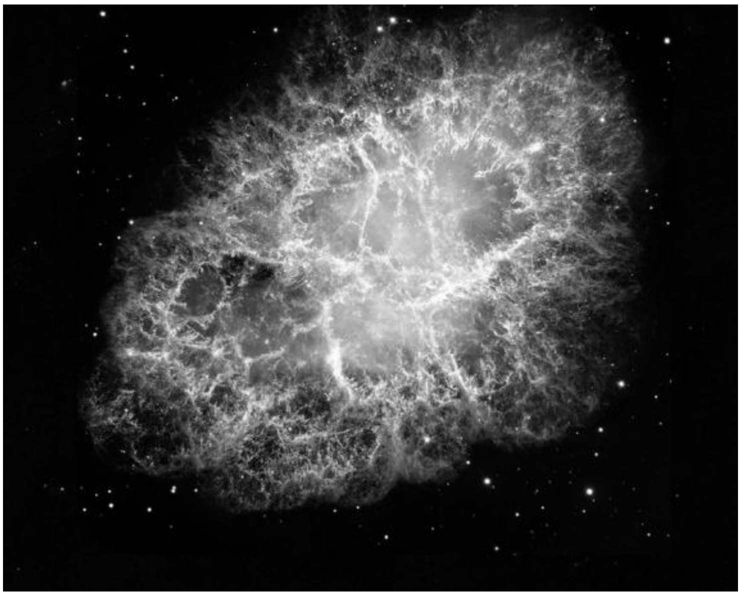

Figure 4.1

Answer：

Figure 4.2

Answer：

Figure 4.3

Answer：

Figure 4.4

Answer:

Instructions

1. **This part of the competition consists of 2 problems (O5 and O6), and is worth a total of 50 points.**
2. **You have 10 minutes to complete these problems.**
3. **Stop working once the timer expires.**
4. **At the end of the examination, hand over the test envelope to the supervisor at the station.**
5. **You will use a telescope to observe five red LED screens at a distance. The telescope is equipped with one eyepiece, but no finderscope. Some of these screens display the names of planets, some show Messier numbers, and some show equatorial coordinates.**

O5. (10 points) The eyepiece's field of view is 45°. Please estimate the field of view (with an accuracy of 0.1 °) of the telescope when observing.

Answer:

O6. (40 points) Write down the text you observed on the screen.

Answers:

Mountain

Instruction

First and foremost, safety should be your main concern！

You can see mountains around the hotel, and in the picture below, the highest peak is marked with an arrow. Estimate the height of this peak relative to the ground of the hotel. You can use everything you can get or make, except climbing up! The team which gives a number closest to the actual answer will win the 'game'.

Give this page to the organizers or volunteers before 23:59, November 8th.

The height: ______________ m; margin of error: _________ m.

Brief description of your process:

No. of Team:

Time of completion (for jury):